**Design a European Option**

* Create a European-style option on the selected stock.
* Clearly specify the type (Call/Put), Strike Price (K) etc.

|  |  |  |
| --- | --- | --- |
| Parameters | Value | Notes |
| Option Type | **Call** | Based on Palantir (PLTR) |
| (Current / Spot Price) | **$25.88** | Using Initial Closing Price from 1 July 2024 |
| (Strike Price) | **$40** |  |
| (Time to Expiry) | **1.00** Years | Makes Calculations for Pricing Simpler |
| (Risk-Free Rate) | **3.93%** | 12-Month Treasury Yield Rate (Bloomberg, 2025) |
| (Annual Volatility) | **70.010%** | As Previously Calculated in Question 1 |
| (Binomial Steps) | **10** Steps | Standard, and keeps Binomial Tree Manageable |

To determine the Strike Price for our new European Option, we have the following choices:

* At-The-Money (ATM) 🡪 Same as our Current Price (K = $25.88)
* Out-Of-The-Money (OTM) 🡪 e.g. K = $40.00 (Only Profitable if Price Continues to Rise)
* In-The-Money (ITM) 🡪 e.g. K = $20.00 (Already has Intrinsic Value)

Therefore, it’s decided the Strike Price, K = $40.00. It is a reasonable value and is slightly out of the money, as the European option chosen is a “Call” rather than a “Put” – indicating that Palantir’s price is expected to rise.

**Option Pricing Using Black-Scholes Model**

Price the option using the **Black-Scholes formula**:

We must first derive the Black-Scholes equation:

**Step 1:** Assume the Stock Follows Geometric Brownian Motion

Where:

* = Current Stock Price
* = Expected Rate of Return on the Stock
* = Constant Volatility of the Stock (Standard Deviation of Returns)
* = Small increment of Time
* = Increment of Standard Wiener Process (Brownian Motion)

This is known as Geometric Brownian Motion (GBM), the standard assumption for asset prices in continuous-time finance.

**Step 2:** Construct a Hedged Portfolio

We consider an option with value , for example, a European Call, and we construct a portfolio made of:

* Long 1 Unit of the Option
* Short Units of the Stock

The goal is to choose such that this portfolio becomes riskless over an infinitesimally small interval.

**Step 3:** Apply Itô’s Lemma to the Option Value

To determine how the option evolves, we apply “Itô’s Lemma” to the function , which depends on both time and the Stochastic Variable :

Where:

* = Sensitivity of Option Value to Time (Theta)
* = Sensitivity of Option to the Stock Price (Delta)
* = Convexity of Option Value (Gamma)

Substituting GBM for gives from **Step 1:**

**Step 4:** Construct Portfolio Change and Eliminate Risk

Substituting and :

Grouping like terms:

Choosing:

This eliminates the stochastic term , making the portfolio risk-free (no exposure to randomness)

**Step 5:** Apply No-Arbitrage 🡪 Risk-Free Return Must Equal

This is because the portfolio is riskless; the return must be equal to the risk-free rate :

However, earlier in **Step 4:**

So, setting the two expressions for equal:

Rearranged:

This is the **Black-Scholes Partial Differential Equation (PDE)**.

**Step 6:** Solve the Black-Scholes PDE with Boundary Conditions

We now solve the PDE using:

* Final Condition (European Call Payoff at Expiry):

Solving the PDE gives the Black-Scholes formula for a European Call Option:

Where:

Where:

* = Price of the European Call Option
* = Current Stock Price
* = Strike Price
* = Time to Expiry
* = Risk-Free Interest Rate
* = Annualised Volatility of the Stock
* = Cumulative Distribution Function (CDF) of the Standard Normal Distribution.

Firstly, we calculate and :

Lastly, we substitute everything into the final equation, the Black-Scholes Call Formula:

Call Option Price = $3.8